9.3: Separable Differential Equations

Entry Task: *(Motivation)*

Implicitly differentiate $x^2 + y^3 = 8$ and solve for $\frac{dy}{dx}$.

Idea: Separate… integrate both sides.

Entry Task continued:

Find the *explicit* solution for

$$\frac{dy}{dx} = \frac{-2x}{3y^2}$$

with $y(0) = 2$.

9.3: Separable Differential Equations

A **separable** differential equation can be written as:

$$\frac{dy}{dx} = f(x)g(y).$$

(or $\frac{dy}{dx} = \frac{f(x)}{g(y)}$ or $\frac{dy}{dx} = \frac{g(y)}{f(x)}$.)

Example: Find the explicit solution to

$$\frac{dy}{dx} = \frac{x}{y^4}$$

with $y(0) = 1.$

Example: Find the explicit solution to

$$\frac{dy}{dx} = \frac{x\sin(2x)}{3y}$$

with $y(0) = -1.$

Example: Find the explicit solution to

$$(x+1)\frac{dy}{dx} = \frac{x^2}{e^y}$$

with $y(0) = 0$.

Law of Natural Growth

Assumption: "*The rate of growth of a population is proportional to the size of the population*."

P(t) = population at year *t*,

$\frac{dP}{dt} =$ rate of change of the population

The law of natural growth assumes

$$\frac{dP}{dt} = kP,$$

for some constant *k*

(we call *k* the *relative growth rate*).

Find the explicit solution to

$$\frac{dP}{dt} = kP \ \text{ with } \ P(0) = P_0$$

1. 500 bacteria are in a dish at $t=0hr$.
8000 bacteria are in the dish at $t=3hr$.
Assume the population grows at a rate proportional to its size.
Find the function, $B(t)$, for the bacteria population with respect to time.

2. The *half-life* of cesium-137 is 30 years. Suppose we start with a 100-mg sample. The mass decays at a rate proportional to its size.
Find the function, m(t), for the mass with respect to time.

3. You invest \$10,000 into a savings account and never make any deposits or withdrawals. The balance grows at a rate proportional to its size (i.e. *interest* is a percentage of the balance at any time). In 3 years, you notice your balance is \$10,400.

Find the function, $A(t)$, for the amount of money in the account with respect to time.